Nordhaus Global Warming Model

Appendix B: Equations of DICE-99 Model

Non-regional

Nordhaus-Boyer [2000]

per capita C, pop

(B.1) $$W = \sum_{t} U[c(t), L(t)]R(t).$$

(B.2) $$R(t) = \prod_{v=0}^{t} [1+\rho(v)]^{-10}$$

$$\rho(t) = \rho(0)\exp(-g^{\rho} t).$$

discounting

(B.3) $$U[c(t), L(t)] = L(t)\{\log[c(t)]\}.$$

(B.4) $$g^{pop}(t) = g^{pop}(0)\exp(-\delta^{pop} t)$$

$$L(t) = L(0)\exp\left(\int_0^t g^{pop}(t)\right).$$

population

damage from temp.

(B.5) $$Q(t) = \Omega(t)(1 - b_1(t)\mu(t)^{b2})A(t)K(t)^{\gamma} L(t)^{1-\gamma}.$$

output

control emissions

(B.6) $$g^{A}(t) = g^{A}(0)\exp(-\delta^{A} t)$$

$$A(t) = A(0)\exp\left(\int_0^t g^{A}(t)\right).$$

tech progress

(B.7) $$\Omega(t) = 1/[1 + D(t)].$$

(B.8) $$D(t) = \theta_1 T(t) + \theta_2 T(t)^2.$$

damage ⇒ $\Omega(t)$

(B.9) $$g^{b}(t) = g^{b}(0)\exp(-\delta^{b} t)$$

$$b_1(t) = b_1(t-1)/(1 + g^{b}(t))$$

$$b_1(0) = b_1{}^{*}.$$

tons/$1000

(B.10) $$E(t) = (1 - \mu(t))\sigma(t)A(t)K(t)^{\gamma} L(t)^{1-\gamma}.$$

energy emissions

(B.11) $$g^{\sigma}(t) = g^{\sigma}(0)\exp(-\delta^{\sigma}{}_1 t - \delta^{\sigma}{}_2 t^2)$$

μ control of emissions

$$\sigma(t) = \sigma(t-1)/(1 + g^{\sigma}(t))$$

$$\sigma(0) = \sigma^{*}.$$

permits

(B.12) $$Q(t) + \tau(t)[\Pi(t) - E(t)] = C(t) + I(t).$$

"subsidy"

(B.13) $II(t) = E(t).$

(B.14) $c(t) = C(t)/L(t).$

(B.15) $K(t) = K(t-1)(1-\delta_K)^{10} + 10 \times I(t-1)$

$K(0) = K^*.$

(B.16) $LU(t) = LU(0)(1-\delta_1)^t$

$ET(t) = E(t) + LU(t).$

(B.17a) $M_{AT}(t) = 10 \times ET(t-1) + \phi_{11} M_{AT}(t-1)$

$-\phi_{12} M_{AT}(t-1) + \phi_{21} M_{UP}(t-1)$

$M_{AT}(0) = M_{AT}^*.$

(B.17b) $M_{UP}(t) = \phi_{22} M_{UP}(t-1) + \phi_{12} M_{AT}(t-1) + \phi_{32} M_{LO}(t-1)$

$M_{UP}(0) = M_{UP}^*.$

(B.17c) $M_{LO}(t) = \phi_{33} M_{LO}(t-1) + \phi_{23} M_{UP}(t-1)$

$M_{LO}(0) = M_{LO}^*.$

(B.18) $F(t) = \eta\{\log[M_{AT}(t)/M_{AT}^{PI}]/\log(2)\} + O(t)$

$O(t) = -0.1965 + 0.13465t \qquad t < 11$

$= 1.15 \qquad t < 10.$

(B.19) $T(t) = T(t-1) + \sigma_1\{F(t) - \lambda T(t-1) - \sigma_2[T(t-1) - T_{LO}(t-1)]\}$

$T(0) = T^*.$

(B.20) $T_{LO}(t) = T_{LO}(t-1) + \sigma_3[T(t-1) - T_{LO}(t-1)]$

$T_{LO} = T_{LO}^*.$